ENTRANCE SCHOLARSHIP EXAMINATION.

QUEENS' COLLEGE. *October*, 1861..

I.

Translate into literal English:

From Καὶ τότ' ἐγὼ τὸν μοχλὸν....
to ....ἔρριψεν ἀπὸ ἕο χερσὶν ἀλύων.

HOM., *Odyss.*

From Στείχων δ' ἱκνοῦμαι τούσδε....
to ....ὤλετ' ἄρ' ἔφυν κακός;

SOPH., *Œd. Tyr.*

From ΣΩΚ. Οὐκ οὖν ἐν τῶν ὁμολογουμένων....
to ....ἀντίλεγε, καί σοι πείσομαι.

PLAT., *Crit.*

From Νόμοι τε πάντες συνεταράχθησαν....
to ....εἰκὸς εἶναι τοῦ βίου τὶ ἀπολαῦσαι.

THUCYD. II.

II.

Translate into literal English:

From C. Canius, eques Romanus nec infacetus....
to ....quid accidisset. Stomachari Canius.

CIC., *Off.*

From Romæ aut circa urbem multa ea hieme..
to ....annos respublica eodem stetisset statu.

LIV.

From Quod si forte preces præcordia....
to ....certe mens mea majus agit.

OVID.

From Quid? cum Picenis excerpens semina....
to ....ignoto discet pendentia tergo.

HOR.

III.

Translate into Greek Prose:

If envy, like anger, did not burn itself in its own fire, and consume and destroy those persons it possesses, before it can destroy those it wishes worst to, it would set the whole world on fire, and leave the most excellent persons the most miserable. Of all the affections and passions which lodge themselves within the breast of man, envy is the most troublesome, the most restless, hath the most of malignity, the most of poison in it. The object she hath an immortal hatred to is virtue; and the war she makes is always against the best and most virtuous men, at least against those who have some signal perfection. No other passion vents itself with that circumspection and deliberation, and is in all its rage and extent in awe of some control. The most choleric and angry man may offend an honest and a worthy person, but he chooses it not; he had rather provoke a worse man, and at worst he recollects himself upon the sight of the magistrate. Lust, that is blind and frantic, gets into the worst company it can, and never assaults chastity. But envy, a more pernicious affection than either of the other, is inquisitive, observes whose merit most draws the eyes of men upon it, is most crowned by the general suffrage; and against that person he shoots all his venom, and without any noise enters into all unlawful combinations against him to destroy him.

Translate into Latin Prose:

Let states that aim at greatness, take heed how their nobility and gentlemen do multiply too fast; for that maketh the common subject grow to be a peasant and base swain, driven out of heart, and, in effect, but a gentleman's labourer. Even as you may see in coppice woods; if you leave your straddles too thick, you shall never have clean underwood, but shrubs and bushes. So in countries, if the gentlemen be too many, the commons will be base; and you will bring it to that, that not the hundredth poll will be fit for an helmet; especially as to the infantry, which is the nerve of an army; and so there will be great population and little strength. This which I speak of hath been nowhere better seen than by comparing of England and France; whereof England, though far less in territory and population, hath been (nevertheless) an overmatch; in regard the middle people of England make good soldiers, which the peasants of France do not: and herein the device of King Henry the Seventh (whereof I have spoken largely in the history of his life) was profound and admirable; in making farms and houses of husbandry of a standard; that is, maintained with such a proportion of land unto them as may breed a subject to live in convenient plenty, and no servile condition; and to keep the plough in the hands of the owners, and not mere hirelings.

IV.

Translate into Latin Hexameters:

While thus he spake, the angelic squadron bright
Turn'd fiery red, sharpening in mooned horns
Their phalanx, and began to hem him round
With parted spears: as thick as when a field
Of Ceres, ripe for harvest, waving bends
Her bearded grove of ears, which way the wind
Sways them; the careful ploughman doubting stands,
Lest on the threshing-floor his hopeful sheaves
Prove chaff. On the other side, Satan, alarm'd,
Collecting all his might, dilated stood
Like Teneriff or Atlas, unremoved.
His stature reach'd the sky, and on his crest
Sat Horror plumed: nor wanted in his grasp
What seem'd both spear and shield. Now dreadful deeds
Might have ensued; nor only Paradise
In this commotion, but the starry cope
Of heaven perhaps, or all the elements
At least had gone to wreck, disturb'd and torn
With violence of this conflict; had not soon
The Eternal, to prevent such horrid fray,
Hung forth in heaven his golden scales.

Translate into Greek Iambic Verse:

Well have ye judged, well ended long debate,
Synod of Gods, and, like to what ye are,
Great things resolved, which, from the lowest deep,

Will once more lift us up, in spite of fate,
Nearer our ancient seat; perhaps in view
Of those bright confines, whence, with neighbouring arms
And opportune excursion, we may chance
Re-enter Heaven; or else in some mild zone
Dwell, not unvisited of Heaven's fair light,
Secure; and at the brightening orient beam
Purge off this gloom: the soft delicious air,
To heal the scar of these corrosive fires,
Shall breathe her balm. But first whom shall we send
In search of this new world? whom shall we find
Sufficient? who shall tempt with wandering feet
The dark unbottom'd infinite abyss,
And through the palpable obscure find out
His uncouth way, or spread his airy flight
Upborne with indefatigable wings
Over the vast abrupt, ere he arrive
The happy isle?

V.—EUCLID.

1. Give Euclid's definitions of a point, a line, and a surface. What other definitions of them have been suggested by geometers? State which you prefer, and for what reasons.

2. If two triangles have two angles of the one respectively equal to two angles of the other, and a side of the one equal to a side of the other, either the sides adjacent to or opposite to those equal angles; the remaining sides and angles are respectively equal to each other.

If the right lines, drawn from the extremities of the base of a triangle, to meet the opposite sides, and making equal angles with the sides, are equal, the triangle is isosceles.

3. Equal triangles, on the same base and on the same side of it, are between the same parallels.

Trisect a triangle by right lines drawn from a point given in one of its sides.

4. In any triangle the square of the side, subtending an acute angle, is less than the sum of the squares of the sides containing that angle, by twice the rectangle contained by either of them and the segment between the acute angle and the perpendicular let fall from the opposite angle.

Given in any triangle the base, the line bisecting the base, and the sum of the sides, construct the triangle.

5. Draw a right line from a given point either without or in the circumference, which shall touch a given circle.

If three circles touch each other externally and the three common tangents be drawn, these tangents shall intersect in a point equidistant from the points of contact of the circles.

6. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If perpendiculars be drawn from any point in the circumference of a given circle to the sides of an inscribed triangle, the points where those perpendiculars cut the sides shall lie in the same right line.

7. Upon a given right line describe a segment of a circle which shall contain an angle equal to a given rectilineal angle.

On the same base and on the same side of it two segments of circles are described, and a point is taken in the one from which chords are drawn to the extremities of the base, find the point in the other from which chords being drawn in like manner, the sum of the chords from the point in the one segment shall be equal to the sum of the chords from the point in the other segment.

8. Describe a circle about a given triangle.

If four right lines intersect each other forming four triangles, the circles circumscribing them shall all pass through one point.

9. Give Euclid's definition of proportional magnitudes.

To what objection has this definition been considered liable?

10. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it shall be similar to the whole triangle and to one another.

Produce a given right line so that the rectangle under it and the produced part shall be equal to a given square.

11. Cut a given finite right line in extreme and mean ratio.

On a given right line construct a right-angled triangle whose three sides shall be in continued proportion.

12. If two right lines meeting one another be paralled to two other right lines which meet one another, but are not in the same plane as the first two, the plane which passes through these is parallel to the plane which passes through the others.

VI.—ALGEBRA.

1. Multiply $a - b$ by $c - d$, where a is greater than b, and c greater than d; and hence deduce the rule of signs in multiplication.

2. Prove the truth of the equation $a^m \times a^n = a^{m+n}$, when m and n are positive integers.

Assuming that the above equation continues true when m and n are negative or fractional, attach a consistent meaning to the expression $a^{\frac{p}{q}}$.

3. Extract the square root of the following quantities:

$1 + x^4 + 6x^3 + 11x^2 + 6x$; ·001, ·1, ·01 to 3 decimal places.

4. Prove a rule for the addition of fractions with different lenominators. Simplify the expression

$$\frac{1}{3(1-x)} + \frac{2+x}{3(1+x+x^2)} + \frac{x}{1+x^3}.$$

5. Shew that the greatest common measure of two compound algebraical quantities is the least common multiple of all the common measures.

6. Solve the equations

$$(1)\ \frac{x-1}{2} - \frac{x-2}{3} + \frac{x-3}{4} = \frac{2}{3}. \qquad (2)\ \frac{x}{x+n} + \frac{x+n}{x} = n.$$

$$(3)\ \frac{x}{a} + \frac{y}{b} - 2\left(\frac{x}{a} - \frac{y}{b}\right) = 7, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - 4\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = 5.$$

7. Eliminate x between the equations

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = m, \quad x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = n.$$

8. A number consisting of three digits, the absolute value of each digit being the same, is 37 times the square of any digit. Find the number.

9. A and B run a mile. At the first heat A gives B a start of 20 yards, and beats him by 30 seconds; at the second heat A gives B a start of 32 seconds, and beats him by $9\frac{5}{11}$ yards. At what rate an hour does A run.

10. Find the sum of n terms of an arithmetic series.

Given the first term and the common difference find n, so that the sum of $2n$ terms may be equal to p times the sum of n terms. Explain the result when $p = 4$, and when $p = 2$.

11. Find the number of combinations of n things taken r together, without assuming the number of variations.

In how many of the combinations do any 3 particular things occur?

12. Prove the Binomial Theorem for a positive integral index.

Find the coefficient of x^n in the expansion of $(1 + x)^{2n}$.

13. Find the number of different ways in which a substance of a ton weight may be weighed by weights of 9 lbs. and 14 lbs.

VII.—TRIGONOMETRY.

1. Shew that the ratio $\frac{\text{arc}}{\text{radius}}$ is a proper measure of an angle.

If the right angle be centesimally divided, and the measure of an angle according to that division be 5·5, where the measure of the right angle is 100, express its magnitude in the common circular measure.

2. Prove the formula $\sin(A+B)=\sin A \cos B + \cos A \sin B$. Hence obtain $\sin 2A$ and $\cos 2A$.

3. Prove the formulæ

$$1 \pm \sin\theta = 2\sin^2\left(\frac{\pi}{4} \pm \frac{\theta}{2}\right), \quad \sin^3\theta\,\frac{\cos 3\theta}{3} + \cos^3\theta\,\frac{\sin 3\theta}{3} = \frac{\sin 4\theta}{4}.$$

4. Prove that in any plane triangle of which a, b, c are the lengths of the sides, and A, B, C the magnitudes of the angles,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Hence deduce, for the triangle, the formula

$$\left(\frac{\sin A + \sin B + \sin C}{a+b+c}\right)^2 = \frac{a\cos A + b\cos B + c\cos C}{2abc}.$$

5. Given in any triangle two sides and the included angle, solve the triangle; and adapt your result to logarithmic computation.

6. Express the area of a triangle in terms of its sides.

In the "ambiguous case" find the difference between the areas of the two triangles.

7. A statue 10 feet high, standing on a column 100 feet high, subtends at the eye of an observer in the horizontal plane, from which the column springs, the same angle as a man 6 feet high standing at the foot of the column; find the distance of the observer from the column.

8. Find the radius of a circle inscribed in a triangle.

Find also the radius of the circle which touches one side and the other two produced.

9. A polygon of n sides is inscribed in a circle; find its area; and the area to which it continually approximates as n is increased without limit.

10. If two circles, the radii of which are a and b, touch each other externally; and if θ be the angle contained by the two common tangents to the circles; shew that

$$\sin\theta = 4\,\frac{(a-b)(ab)^{\frac{1}{2}}}{(a+b)^2}.$$

11. Prove Demoivre's Theorem; and apply it to find the 3 values of $(-1)^{\frac{1}{3}}$.

12. Expand $\sin\theta$ in terms of θ; and prove that

$$2\cos\theta = \varepsilon^{\theta\sqrt{(-1)}} + \varepsilon^{-\theta\sqrt{(-1)}}.$$

13. Prove Machin's series for the value of π.

14. Find the sum of the series

$$\cos\frac{\pi}{n} + \cos\frac{2\pi}{n} + \cos\frac{3\pi}{n} + \ldots + \cos\frac{(2n-1)\pi}{n}.$$
